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Indexation in Long Term Labor Contracts:

A Theoretical and Empirical Analysis

David Card

Graduate School of Business

University of Chicago

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Introduction

The importance of wage indexation for the persistence of inflation and the macroeconomic performance of the economy has been widely acknowledged.¹ At the same time, recent collective bargaining developments have drawn attention to the role of cost of living escalation in the wage determination process.² In both instances, public and professional interest has focused on the response of indexed wage rates to concurrent price increases.³ On the other hand, the characteristics of this response have only recently been investigated. The results of this research indicate that labor contracts differ widely in the size of escalated wage increases

¹For example, Fischer[12], Gray[14], and Guikerman[9] have emphasized the positive and normative consequences of alternative indexation regimes in the presence of real and monetary disturbances in the economy. Taylor[22] has demonstrated the important role of indexation provisions in determining the path of the economy between high- and low-inflation regimes.

²Recently, the United Auto Workers have deferred index linked wage increases in many of their contracts. Similar arrangements were made between the United Rubber Workers and Uniroyal in June 1981. Interestingly enough, the 1982 round of contracts in the rubber industry have retained cost of living escalation and eliminated non-contingent increases entirely.

³In fact, as early as the Korean War, public policy concentrated on this aspect of escalated contracts. Under the War Stabilization Board, indexation clauses were legally restricted to average elasticities of indexation less than unity.

awarded for each increase in consumer prices.⁴ Furthermore, a major fraction of the dispersion in the response of index-linked wage increases is attributable to the industry of origin of the contract.

These two facts suggest that contractual wage indexation provisions cannot be explained by a simple model of real wage insurance.⁵ The starting point for this paper is the recognition that parties to a collective bargaining agreement are typically concerned with a wide range of relative prices, including the price of the firm's inputs and outputs and the level of wages elsewhere in the economy. Unlike the consumer price index, however, many of these prices are private information and not freely available to one party or the other. For this reason, contracts written contingent on relative prices are potentially costly to administer and monitor.⁶ However, given the structure of aggregate and relative prices in the economy, observations on the aggregate price index convey information on the levels of relative prices. In light of this information, the role of the contractual escalation formula is to incorporate inferences of the unobserved relative prices into the wage adjustment process.

⁴For base wage rates in a sample of Canadian contracts from 1968-1975, the elasticity of wages with respect to prices during the course of indexation ranged from .52 to 1.50. See Card[8].

⁵For example, the model of Azariadis[3] predicts a constant real wage rate over the life of the contract.

⁶Labor contracts with wage rates indexed to product prices were written in the copper industry in the interwar period, and in the iron industry in the 1870's. See Taft[21]. Since the important indexed agreement between General Motors and the United Automobile Workers in 1948, wage rates have been linked to consumer prices exclusively.

According to this interpretation, the response of contractual wage rates to changes in the aggregate price index depends on two factors: the extent to which signals on relative prices can be extracted from aggregate prices, and the extent to which it is desirable to adjust real wages to these signals. A strong test of the theory is obtained by comparing the predicted and actual distributions of elasticities of escalated wage increases with respect to consumer prices across contracts.

In the first section of the paper, a simple model of contractual wage indexation is developed. The model is summarized by an expression for the elasticity of the contractual wage escalator with respect to prices. The implications of two alternative assumptions on the form of union preferences are catalogued and related to earlier work by Blanchard[4]. In the second section of the paper, the model is tested against a sample of contracts from the Canadian manufacturing sector. An interpretative link is developed between the model and the form of typical contractual escalation provisions. This link permits tests of the predictive content of the theory, and tests of alternative specifications of the model. The implications of the empirical analysis for the interpretation of observed contractual arrangements are discussed in the concluding section of the paper.

I. A Model of Long Term Indexed Contracts

(a) General Considerations

The analysis of indexed labor contracts in this paper is based on a model of bilateral monopoly with imperfect information. Bargaining takes place between a firm and a homogeneous collection of workers with specific skills and training. In each period, these parties determine the levels of wages, employment and non-labor inputs, taking as exogenous an output demand shock, non-labor input prices, alternative real wages, and the level of aggregate prices. Information on aggregate prices is assumed to be freely available to both workers and the firm, while information on output market conditions and input prices is available only to the firm, and information on alternative wages is freely available only to workers. It will also be assumed that in the absence of asymmetric information, workers and the firm would be in agreement as to the choice of wages, employment and non-labor inputs. With these assumptions, formal contract negotiations are interpreted as the process of information exchange between the contacting parties. Since both sides will generally have an incentive to conceal their private information, contract negotiations are potentially costly. In this situation, multiperiod contracts arise naturally as a means of reducing negotiation costs, although at the expense of some efficiency in the allocation of earnings and employment.

Under a multiperiod contracting arrangement, complete information is exchanged only occasionally (once every two or three years). In the

intervening periods, adjustments to wages and employment are made through incentive compatible schemes that take account of the underlying asymmetry in information. An important class of these schemes has the feature that one party (usually the better informed) is given the contractual right to act on behalf of the pair, while payments between the parties are made contingent on variables observed by both.⁷ Labor contracts in which the wage rate is indexed to the aggregate price level and employment is determined by the firm's labor demand schedule are particularly simple examples of this kind of arrangement.

Recently, a number of authors have argued that a natural variable on which to base the compensation of workers is the level of employment at the firm.⁸ Since the employment decision reveals at least some information on the state of demand and costs of materials observed by the firm, employment-contingent contracts can help to resolve the information asymmetry between workers and the firm.⁹ Nonetheless, the employment contingent component of compensation in typical labor contracts is

⁷A discussion of incentive-compatible schemes in a labor market context is presented in Hall and Lazear[17].

⁸See for example Calvo and Phelps[5], Hall and Lilien[18], and Grossman and Hart[16].

⁹Hall and Lilien[18] show that in the absence of uncertainty about alternative wages, allowing for ex ante transfers between workers and the firm, and assuming risk neutrality on the part of the owners of the firm, employment-contingent wage schedules achieve ex-post efficient employment allocations. Grossman and Hart[16] demonstrate that relaxing the assumption of risk neutrality of owners leads to a conflict between the risk-sharing and resource allocation goals of the contract that cannot be completely resolved by a simple employment-contingent wage schedule.

restricted to individual-specific overtime payments. For convenience then, and in the absence of empirical evidence to the contrary, the aggregate price index is assumed to be the only information shared by workers and the firm on which intracontract contingencies can be based. Furthermore, intracontract employment and input decisions are assumed to be made unilaterally by the firm, acting as an agent for its workers. This assumption is consistent with the failure to observe contractually specified employment levels (or index-linked employment schedules) in typical North American labor contracts.

(b) The Model

Let p_t represent the aggregate price level in period t , θ_t a real shock in the firm's output demand, r_t the real price of non-labor inputs, a_t the alternative real wage available to workers, w_t the contractual nominal wage rate, M_t the level of non-labor inputs, and L_t the level of employment. Denote the revenue function of the firm by $R(L, M, \theta)$. Real profits in period t are given by:

$$R(L_t, M_t, \theta_t) - w_t \cdot L_t / p_t - r_t \cdot M_t \quad .$$

Assume that the preferences of the firm's owners over alternative profit streams are represented by a function of the form:

$$(1) \quad \sum_t E (v\{R(L_t, M_t, \theta_t) - (w_t/p_t)L_t - r_t M_t\}),$$

where $v\{ \}$ is a concave and increasing utility function and E denotes

expectations based on information at $t=1$. Assume that in each period, a selection L_t of the original L^0 employees of the firm are retained at the firm, while the remainder obtain employment at the alternative real wage rate. The value of a contract with the firm for a typical employee is the value of a lottery that yields the contract wage with probability L_t/L^0 , and the alternative wage with probability $1-L_t/L^0$.¹⁰ Let $u\{\cdot\}$ be an increasing and concave utility function and assume that employees' preferences can be represented by a function of the form:

$$(2) \quad \sum_t E (L_t/L^0 u\{w_t/p_t\} + (1-L_t/L^0)u\{a_t\}).$$

The preference representations (1) and (2) imply additive separability and no borrowing or lending on the part of owners or workers. The analysis of escalation provisions is unchanged with unrestricted borrowing and lending, provided that fully indexed savings instruments are available. However, as Blinder[5] has observed, when indexed bonds are not available, wage indexation provisions are potentially influenced by the desire to insure the real value of employees' or employers' savings. Since the focus of this analysis is on the sources of inter-industry differences in escalation provisions, savings will be ignored.

In this setting, the optimal contract length equates the expected costs of renegotiation with the expected benefits from the exchange of

¹⁰The alternative real wage may include unemployment benefits, the value of leisure, and/or wages in another job. Hours per worker are taken as fixed.

complete information between the two parties.¹¹ For simplicity, however, escalation provisions will be analysed conditional on a two period contract. The analysis of wage indexation in each period of a longer contract is analogous.

The level of second period employment is determined by the firms's demand for labor, subject to the constraints that $L_2 \leq L^0$ and that L_2 workers are actually available to work at the wage w_2 . Both constraints will be ignored in this development.¹² Let

$$\Pi(\theta, r, w, p) = \max_{L, M} R(L, M, \theta) - wL/p - rM$$

represent the profit function of the firm. For notational convenience, time subscripts will be dropped, and all variables will be considered as dated at $t=2$. Given the preference representations (1) and (2), the contractual wage function for second period wages solves the following program:

¹¹See Gray[15] for a complete analysis of contract length determination.

¹²In the presence of significant relative wage effects of unionization, the impact of ignoring these constraints is likely to be small.

$$(3) \quad \max \quad E \left(u\{a\} + (L/L_0)(u\{w/p\} - u\{a\}) + hv\{\Pi\} \right) \\ w(p) \\ \text{s.t. } L = -1/p \delta \Pi / \delta w,$$

for some fixed positive number h . If lump sum transfers between workers and the firm are available, then $h=1/L^0$, reflecting the tradeoff of one unit of owner's consumption for $1/L^0$ units of consumption per employee. In the absence of such transfers, $h \leq 1/L^0$, with strict inequality whenever distributional goals conflict with efficiency in determining the wage schedule.

The first order condition for (3) requires that at each p , the expected marginal utilities of workers and owners are proportional. To simplify notation, let

$$U(w,p,a,L) = u\{a\} + (L/L^0)(u\{w/p\} - u\{a\}).$$

The optimal escalator function $w(p)$ satisfies the first order condition

$$(4) \quad E \left(\frac{\delta U}{\delta w} + h \frac{\delta v\{\Pi\}}{\delta w} \mid p \right) = 0$$

at every realization of second period prices. Let $f(\theta,r,a|p)$ denote the conditional density of (θ,r,a) given p . Assume that $f(\cdot|p)$ is differentiable with respect to p and that the range of (θ,r,a) is independent of p . Treating (4) as an identity in p for a particular solution function $w(p)$ and fixed h , the derivative of $w(p)$ is given by:

$$(5) \quad \frac{dw}{dp} - \frac{w}{p} = \frac{1}{D} \iiint \left(\frac{\delta U}{\delta w} + h \frac{\delta v\{\pi\}}{\delta w} \right) \frac{df}{dp} (\theta, r, a | p) da dr d\theta$$

where

$$D = -E \left(\frac{\delta^2 U}{\delta w^2} + h \frac{\delta^2 v\{\pi\}}{\delta w^2} \mid p \right) > 0.$$

The wage escalator is greater than or less than unit elastic at the price level p as the right hand side of (5) is positive or negative.

If the aggregate price level conveys no information on the unobserved real variables, the conditional density $f(\theta, r, a | p)$ is independent of p , and the right hand side of equation (5) is zero. In that case, the optimal wage escalation function maintains a constant real wage and the elasticity of indexation of nominal wages with respect to prices is unity. Otherwise, the conditional distribution of (θ, r, a) depends on prices and the real wage that satisfies (4) is a function of p . There are two reasons for real wages to respond to aggregate prices. First, since employment is set unilaterally by the firm, the real contractual wage rate may adjust with prices to achieve different levels of contractual employment in accordance with the different inferences of (θ, r, a) . Second, if workers and/or owners are risk averse, the real wage rate may respond with prices to stabilize the real incomes of workers and/or owners across different price realizations. To sort out these two influences, it is useful to place some further restrictions on the structure of technology, preferences, and uncertainty.

In this model, the nature of uncertainty is summarized by the joint distribution of p , θ , a , and r . Assume that these variables have a joint lognormal distribution with:

$$\begin{aligned} E(\log p) &= \log \bar{p} \quad , \quad E(\log \theta) = 0, \\ E(\log a) &= \log \bar{a} \quad , \quad E(\log r) = 0, \\ \text{var}(\log p) &= \sigma_p^2 \quad , \quad \text{var}(\log \theta) = \sigma_1^2, \\ \text{var}(\log r) &= \sigma_2^2 \quad , \quad \text{var}(\log a) = \sigma_3^2, \\ \text{cov}(\log p, \log \theta) &= \rho_1 \sigma_p \sigma_1, \\ \text{cov}(\log p, \log r) &= \rho_2 \sigma_p \sigma_2, \\ \text{cov}(\log p, \log a) &= \rho_3 \sigma_p \sigma_3. \end{aligned}$$

The preference representations (1) and (2) are conveniently parameterized by assuming that both $u\{ \}$ and $v\{ \}$ are of the constant relative risk aversion class. In particular, suppose that

$$\begin{aligned} u\{x\} &= 1/(1-d) x^{1-d} \quad , \quad d > 0 \quad , \\ \text{and} \\ v\{x\} &= 1/(1-\gamma) x^{1-\gamma} \quad , \quad \gamma > 0 \quad , \end{aligned}$$

where the constants d and γ are the indexes of relative risk aversion of workers and owners respectively. Finally, to obtain closed form expressions, it is convenient to consider profit functions of the form:

$$\Pi(\theta, r, w, p) = (w/p)^{c_1} \theta^{c_2} r^{c_3},$$

parameterized by the constants c_1 , c_2 , and c_3 . This functional form is consistent with a Cobb-Douglas technology and implies constant elasticities of labor demand with respect to wages, input prices, and the output market shock.¹³

c) Risk Neutrality

The case of risk neutrality on the part of both owners and workers is of special interest because it represents the one most thoroughly analysed in the literature on contracting and indexation.¹⁴ If $d=0$ then

$$U = a + (L/L^0) (w/p - a),$$

and workers' preferences parallel those of a union that maximizes its excess wage bill.¹⁵ The first order condition (4) becomes

$$(4a) \quad \iint L/p \int (1 - \beta p/w(w/p-a) - hL^0) f(a|p) da f(\theta, r|p) d\theta dr = 0,$$

where $-\beta < -1$ is the elasticity of labor demand with respect to real wages. The marginal utilities of both owners and workers are proportional to

¹³For profits to be positive and decreasing in w and r it is necessary to restrict the own price elasticities of factor demand to exceed unity, and the cross-price elasticities to be negative.

¹⁴Risk neutrality is implicit in the models of Gray[14], Blanchard[4], and Hall and Lilien[18].

¹⁵This model of union behaviour is generally associated with Dunlop[11].

employment and the necessary conditions for optimality are satisfied if

$$(6) \quad w/p = E(a|p) \beta / (hL^0 - 1 + \beta) .$$

If $hL^0 = 1$, the contract maximizes the sum of profits and the utility of workers, measured in money units. In that case, $w/p = E(a|p)$, and the contract wage equals the expected opportunity wage at each realization of prices. Bargaining outcomes that distribute positive rents to employees are those with $hL^0 < 1$. For such contracts, the wage escalator maintains a constant proportional markup over the expected opportunity wage.

Differentiating (6) with respect to p and using the fact that

$$E(a|p) = \tilde{a} \exp\{ r_2 \log(p/\tilde{p}) + \frac{1}{2} \sigma_2^2 (1 - \rho_2^2) \} ,$$

where $r_2 = \rho_2 \sigma_2 / \sigma_p$ is the population regression coefficient of innovations in the logarithm of alternative real wage rates on innovations in the aggregate price index, we obtain the following expression for the elasticity of indexation:

$$(7) \quad \frac{dw}{dp} \frac{p}{w} = 1 + r_2 .$$

In the absence of risk aversion, contractual wages respond only to the inference of alternative wages. Furthermore, real contractual wages are unit-elastic with respect to the inference of a .

The conclusion that indexation provisions depend only on the relationship between aggregate prices and opportunity wages is perhaps

surprising, in light of Blanchard's[4] emphasis on the correlation of raw materials prices with aggregate prices as a determinant of the elasticity of indexation. At this stage it is worth indicating how these different conclusions arise, and also how the present analysis is related to the problem of choosing a wage-employment schedule to achieve efficiency when the firm sets employment unilaterally.

To generalize the discussion somewhat in the specific context of risk neutrality, suppose that the net benefit of the contract to all L^0 workers collectively is given by:

$$(w/p)L - c(L,a) ,$$

where $c(L,a)$ has the interpretation of an opportunity cost function, associated with the supply of L units of labor to the firm when the alternative wage is a . The value of the contract to any single worker is a fraction $1/L^0$ of this total benefit. The Dunlop preference representation is obtained by assuming that the opportunity cost function is $(L - L^0)a$, in which case the marginal cost of increasing employment at the firm is a . The contract allocates labor efficiently ex post if expected marginal revenues and costs of additional employment are equal, conditional on the observed level of aggregate prices. As Hall and Lilien[18] have indicated, efficiency can be achieved in a contract with unilateral employment determination by presenting the firm with a marginal cost of employment schedule that mimics the actual schedule $\delta c/\delta L$. In particular, consider a wage schedule $w(p)$ such that

$$(8) \quad \frac{w}{p} = E\left(\frac{\delta c}{\delta L} (L, a) \mid p \right) .$$

Since the firm sets the wage rate equal to the marginal revenue product in each realization of p, θ , and r , ex post efficiency is assured by (8).

In the absence of distributional requirements, the wage indexation function is determined on the basis of allocative efficiency, and the contractual escalator is characterized by (8). For the Dunlop utility function, the marginal cost of employment is a , independent of L , and the elasticity of indexation is given by (7). On the other hand, Blanchard[4] has considered preferences such that the marginal cost of employment is increasing in L . Specifically, suppose that $\delta c / \delta L = a^\phi L^{1/\epsilon}$. To reflect the marginal cost of contractual employment, the wage function must satisfy

$$\frac{w}{p} = E\left(a^\phi L^{1/\epsilon} \mid p \right)$$

at each price level. Substituting for employment from a labor demand schedule of the form

$$\log L = -\beta \log(w/p) - \eta \log r + \mu \log \theta ,$$

where η is the cross elasticity of labor demand with respect to non-labor input prices and $\mu > 0$ reflects the impact of output market shocks on labor demand, the contractual wage rate can be expressed in terms of the conditional inferences of θ and r , given p .

Differentiating this expression with respect to p yields:

$$(7a) \quad \frac{dw}{dp} \frac{p}{w} - 1 = \frac{\mu}{\beta + \epsilon} r_1 - \frac{\eta}{\beta + \epsilon} r_2 + \frac{\phi \epsilon}{\beta + \epsilon} r_3$$

where $r_1 = \rho_1 \sigma_1 / \sigma_p$ and $r_2 = \rho_2 \sigma_2 / \sigma_p$ are the population regression coefficients of innovations in θ and r respectively on innovations in p . If $\phi = \mu = 0$ this expression reduces to the one given by Blanchard. As $\epsilon \rightarrow \infty$ with $\phi = 1$, the opportunity cost function becomes infinitely elastic at the real wage rate a and equation (7) is obtained in the limit. If $\epsilon < \infty$, the real contractual wage established by the escalation function depends on the inferences of all three unobserved variables. Since the opportunity cost of employment is increasing, as θ increases or r falls and employment demand increases, the real contract wage must increase.

In a risk neutral setting, the importance of firm specific input and output market conditions for the slope of the wage escalator depends on the opportunity cost of employment function. On the other hand, the elasticity of the indexation schedule is independent of distributional considerations in the absence of risk aversion. For the case of a constant marginal cost of employment, this result is illustrated by equations (6) and (7). In fact, however, an analogous result holds for more general marginal cost functions and is proved in Appendix A. With risk neutrality, the contractual wage schedule maintains a constant proportional markup over the expected marginal cost of employment, and the elasticity of indexation is

given by (7a), independent of the value of h in equation (4). In the absence of risk aversion, the issue of competing distributional and allocative goals is irrelevant for the elasticity of the wage escalation function.

d) Risk Aversion

Risk aversion introduces an additional role for the contractual wage escalator: that of stabilizing the real incomes of workers and owners across different realizations of prices. As an illustration, suppose that workers are fully tenured. With a fixed labor force, the allocative role of the contractual wage is eliminated and insurance considerations determine the wage escalator function. If owners are risk neutral, the escalator is unit-elastic and workers' real earnings are fully insured across states.¹⁶ On the other hand, when owners are risk averse, the wage function can provide insurance from workers to owners. If the correlation of output demand shocks with innovations in p is positive, as aggregate prices rise the demand for the firm's output increases, and the wage escalator will be greater than unit-elastic in order to stabilize the profits of owners. Conversely, in a counter-cyclical industry, higher aggregate prices signal a negative demand shock, and the wage escalator will be less than unit elastic.¹⁷

¹⁶This is an implication of the model of Azariadis[3].

¹⁷These assertions can be proved easily by examining the elasticity condition (5) with $\Pi = \theta \cdot (w/p)L$, and $U = u(w/p)$.

With risk aversion on the part of owners and workers, the derivation of the wage indexation schedule is straightforward but somewhat tedious. Details are presented in Appendix B. The first order condition (4) can be differentiated to yield:

$$(9) \quad \frac{dw}{dp} \frac{p}{w} = 1 + \frac{1}{D'} \beta r_3 + \frac{1}{D'} \mu \gamma \left\{ \frac{\beta}{1-d} + \frac{1-d-\beta}{(1-d)\psi(p)} \right\} r_1 - \frac{1}{D'} \eta \gamma \left\{ \frac{\beta}{1-d} + \frac{1-d-\beta}{(1-d)\psi(p)} \right\} r_2$$

where

$$D' = \frac{(d-\gamma(1-\beta))(1-d-\beta)}{(1-d)\psi(p)} + \beta \frac{1-\gamma(1-\beta)}{(1-d)}$$

and

$$\psi(p) = (w/p)^{1-d} E(a^{d-1} | p) .$$

Inspection of (9) reveals that the wage indexation function is independent of r_1 and r_2 when owners are risk neutral ($\gamma = 0$). In that case, both the allocative and insurance roles of the wage function depend only on the conditional inference of the opportunity wage schedule.

In general, the wage escalator defined by equation (9) does not exhibit a constant elasticity of indexation. However, for $\log p$ in a neighborhood of its expected value, if the elasticity of indexation at each price realization is not far from unity, (9) is well approximated by a constant elasticity wage schedule. Let \tilde{w} equal the expected nominal wage rate at the start of indexation. Define $z = (\tilde{w}/\tilde{p})/\tilde{a}$, and observe that $\psi(p)$

$\approx z^{1-d}$ if σ_3 is small.¹⁸ The elasticity of indexation is approximately:

$$(10) \quad \frac{dw}{dp} \frac{p}{w} = 1 + c_1 r_1 + c_2 r_2 + c_3 r_3 \quad ,$$

where

$$c_1 = \mu \gamma g / D^* \quad ,$$

$$c_2 = -\eta \gamma g / D^* \quad ,$$

$$c_3 = \beta / D^* \quad ,$$

and

$$g = 1/(1-d) \{ \beta + (1-d-\beta)z^{1-d} \} \quad ,$$

while

$$D^* = \beta \frac{1+\gamma(\beta-1)}{1-d} + \frac{1-d-\beta}{1-d} (d+\gamma(\beta-1))z^{1-d} \quad .$$

If $\gamma=d=0$, then $D^* = \beta$, $c_3 = 1$, $c_1 = c_2 = 0$, and (10) reduces to (7). The denominator term D^* is necessarily non-positive if $w(p)$ is a solution to the program (3) at the price level p . A sufficient condition is $z < 1 + \zeta$ for some positive number ζ . The expression g is positive under similar conditions.

¹⁸Altonji and Ashenfelter[1] estimate an individual-specific real wage variance (controlling for fixed effects) of approximately 0.11.

The elasticity of real contractual wages with respect to changes in the inferences of the unobserved variables (θ, a, r) depends on the magnitude of the coefficients c_1 , c_2 , and c_3 . Comparative statics results for these three coefficients can be obtained for z in a neighborhood of unity. For instance, with z close to one, c_1 is decreasing in z while c_3 is increasing in z . In contracts with a higher relative wage, greater emphasis is placed on stabilizing the differential between the contract wage and the alternative wage. On the other hand, increases in the relative risk aversion of owners imply a greater emphasis on stabilizing real profits. Since profits are independent of the alternative wage, but depend critically on θ and r , increases in γ are associated with an increase in the absolute values of c_1 and c_2 , and a decrease in the absolute value of c_3 .

The effect of changes in either d or β on the characteristics of the wage indexation rule are ambiguous. However, for reasonable values of d , β , and z , the coefficient c_3 is increasing in β and decreasing in d , whereas the absolute values of the coefficients c_1 and c_2 are decreasing in β and d .¹⁹ If β is higher, contractual employment and real profits are more sensitive to the contractual wage. On the one hand, to stabilize profits it is desirable to increase the responsiveness of contractual wages to inferences of θ and r , and reduce the responsiveness of contractual wages

¹⁹We have treated μ as a third parameter in the profit function of the firm. If the firm is competitive in its output market, θ is interpreted as the relative selling price of output and $\mu = \eta + \beta$. In that case, the coefficient c_1 is also decreasing in both d and β .

to inferences of a . On the other hand, to stabilize workers' utilities it is desirable to de-emphasize inferences of θ and r , and increase the sensitivity of contract wages to inferences of the alternative wage. For reasonable choices of the parameters, however, the latter considerations appear to dominate.

e) Summary of the Model

The model of indexation delivers an expression for the elasticity of contractual wages with respect to prices in terms of the parameters of preferences, production, and the stochastic environment facing the contracting parties. At this stage, it is instructive to compare the implications of the two versions of the model, represented by equations (7a) and (10) respectively. The form of both expressions is similar: in each case the deviation of the elasticity of indexation from unity is a weighted average of the parameters r_1 , r_2 and r_3 . Furthermore, both (7a) and (10) place the same non-linear restriction between the coefficients of r_1 and r_2 .²⁰ Other than functional form differences, the major empirical distinction between the two is in the role of z , the average or expected relative wage effect of unionization.²¹ As noted earlier, in the absence of risk aversion, the elasticity of the wage escalation function is

²⁰The ratio of the coefficient of r_1 to the coefficient of r_2 is $-\mu/\eta$.

²¹Treating ϕ , ε , γ and d as unobservable parameters.

independent of any distributional considerations in the contract. This presents the possibility of testing an 'increasing marginal cost of employment' model of indexation against a 'risk sharing' model by testing for the importance of the relative wage effect of unionization in determining the elasticity of indexation.

Some idea of the predictive content of the model can be obtained by considering the implications of (7a) or (10) for given values of the underlying parameters. Aggregate time series evidence suggests that real wage and price innovations are negatively correlated.²² This is consistent with the casual observation that in many jobs, nominal wages are adjusted only once or twice a year. In addition, unemployment and social security benefits are not fully indexed to contemporaneous price increases. Therefore, from the point of view of any particular bargaining group, the coefficient r_3 is likely to be negative. Furthermore, one would expect r_3 to be more negative for workers whose job opportunities are concentrated in sectors characterized by non-indexed nominal wage contracting, and less negative for workers whose job opportunities extend to labor markets with auction characteristics.

The coefficients r_1 and r_2 can be expected to vary across industries, according to the nature of the inputs and outputs of the industry. At least in recent years, the correlation of innovations in real raw materials

²²Using residuals from autoregressions on annual data from Canada, the correlation of innovations in average real wages in manufacturing and the C.P.I. is -0.41. Similar results are obtained using quarterly data from Canada, and quarterly data from the U.S.

prices and the consumer price index has been positive.²³ This implies higher values of r_2 in industries whose inputs consist largely of unprocessed goods and materials. On the other hand, it is often argued that nominal industry prices are sticky, at least among manufacturing industries.²⁴ On the basis of this argument, one would expect lower values of r_2 in those industries whose inputs consist of the processed outputs of other industries. By the same token, it seems reasonable to expect that the correlation of innovations in real industry selling prices and the consumer price index is higher among non-durable manufacturing industries and service industries, and lower among durable manufacturing industries.

On the basis of this pattern for the parameters r_1 , r_2 , and r_3 , the model of indexation predicts higher elasticities of indexation in service industries and non-durable (particularly food and textiles) manufacturing industries, and lower elasticities of indexation among contracts in the durable manufacturing industries.²⁵ Furthermore, given a negative value of r_3 , elasticities of indexation might be expected to average somewhat less

²³Recent data are dominated by the simultaneous increases that occurred in the real prices of energy, food, and raw materials, together with the aggregate price index, over the 1973-75 period.

²⁴The early literature on this hypothesis is summarized by Stigler and Kindahl[20], who provide some evidence against the sticky price notion.

²⁵Observe that both (7a) and (10) imply that the coefficient of r_1 is larger in magnitude than the coefficient of r_2 , at least for price-taking industries ($\mu = \beta + \eta > \eta$). Thus among industries with $r_1 = r_2$, and positive r_1 and r_2 , average elasticities of indexation are predicted to be relatively high.

than unity across all industries. These implications are generally consistent with the pattern of measured elasticities of indexation presented in Card[8]. The extent to which the model of this paper can describe the distribution of index-linked responses, taking account of both the industry distribution of r_1 , r_2 , and r_3 , and the pattern of technology across industries, is pursued more formally in the next section.

II. Empirical Performance of the Model

a) General Considerations

The model of wage indexation developed in the previous section gives rise to an expression for the elasticity of the contractual wage escalator in terms of the parameters of preferences, technology, and the stochastic structure of real and aggregate prices. In contrast to the escalation schedule described by the model, however, observed wage indexation provisions typically fail to provide a fixed percentage increase in wages for each percentage increase in the consumer price index (C.P.I.).²⁶ Furthermore, escalated labor contracts generally include both contingent and substantial non-contingent wage increases.²⁷ Finally, in many labor

²⁶See Card[8]. Over 90 percent of the contracts analysed there feature non-proportional wage indexation to the CPI.

²⁷See Table 2 of Card[8]. Non-contingent increases account for about 70 percent of the total realized wage increases in the sample. The size of non-contingent increases among escalated contracts is typically smaller

contracts, escalation provisions are shared among a wide range of employees, including both skilled and unskilled production workers. This sub-section develops an interpretative link between the simple model of wage indexation described by (7a) or (10) and the form of contractual escalation provisions. The major features of this link include recognition of the role of non-contingent wage increases in escalated wage contracts, and consideration of the heterogeneity of the contractual labor force in modelling shared wage escalation clauses.

It has been observed that among indexed contracts, the fraction of wage increases attributable to contingent as opposed to non-contingent wage increases is small, but increasing in periods of unanticipated inflation.²⁸ This suggests that indexation provisions in many labor contracts are written to provide marginal adjustments to contractual wages as the aggregate price deviates from its expected level. On the other hand, in some contracts, non-contingent wage increases are small and a majority of wage adjustment is achieved through index-linked wage increases. In the framework of a two period model, contracts differ by the extent to which the expected level of second period aggregate prices is incorporated into the non-contingent component of second period wages. To pursue this

than among non-escalated contracts. For major agreements in the U.S. in 1982, annualized non-contingent increases averaged 9.2 percent among non-escalated contracts, and 3.7 percent among escalated contracts. See LeRoy[19].

²⁸For example, Table 2 of Card[8] indicates that the share of contingent wage increases in total increases averaged 31 percent over all contracts. However, this share was 41 percent among contracts signed in 1971, many of which spanned the period of rapid price increases in 1973 and 1974.

formally, suppose that (10) (or (7a) by a suitable redefinition of the coefficients c_1 , c_2 , and c_3) describes the desired relation between second period wages and prices in a relevant range of prices. Integrating (10), we obtain the following expression for second period wages:

$$\log w - \text{constant} = e^* \log p,$$

where $e^* = 1 + c_1 r_1 + c_2 r_2 + c_3 r_3$. Suppose that the non-contingent component of second period wages, w^Ω , incorporates the constant term and some fraction of the unconditional expectation of the right hand side of this expression:

$$\log w^\Omega = \text{constant} + f \cdot e^* E[\log p]$$

The contractual wage rate, conditional on a second period aggregate price p , can be written as

$$(11) \quad \log w = \log w^\Omega + e^* \{ \log p - \log \bar{p} \},$$

where $\log \bar{p} = f \cdot E[\log p]$. In this expression, the desired second period wage is achieved through a combination of the non-contingent wage w^Ω , and contingent increases based on the excess of p over \bar{p} . Larger non-contingent increases are associated with a longer delay in the operation of escalation provisions. Provided that f is small enough, the contingent component of wage change will be positive with arbitrarily high probability.²⁹

²⁹Contingent wage changes are not restricted to be positive in all contracts. For example, the first escalated contract signed by General Motors and the United Automobile Workers actually reduced nominal wages in

This development highlights the tradeoff between contingent and non-contingent wage increases, and illustrates the difficulty in interpreting ex post contingent wage increases in isolation from the non-contingent increases that accompany them. On the other hand, inspection of equation (11) reveals that the marginal elasticity of nominal wages with respect to prices is an appropriate estimator of the ex ante elasticity e^* , regardless of the size of non-contingent wage increases in the contract. Although contractual escalators rarely take the form of (11), a useful interpretation of observed provisions is that they represent piece-wise linear approximations to the constant elasticity wage-price trajectories described by (11). Under this interpretation, the marginal elasticity of the observed (non-proportional) wage indexation schedule, measured at the start of indexation, is likely to be a good estimator of the marginal elasticity of the "desired" wage schedule (11). This interpretation of non-proportional escalation formulas will be adopted here, and equations (7a) and (10) will be viewed as alternative descriptions of the marginal elasticity of indexation, as measured at the start of escalation.³⁰

In a majority of indexed contracts the cost of living escalation clause specifies an absolute wage increment for each point increase in the C.P.I. Typically, the same increment is applied to every wage rate in the

the later years of the contract. See Garbarino[13].

³⁰As an empirical matter, in contracts with non-proportional wage escalation provisions (which provide a fixed absolute wage increment for each fixed absolute increment in prices), the marginal elasticity of indexation is relatively constant during the course of indexation.

contract. Let α represent the contractually specified derivative of wages with respect to the C.P.I., let $E \log w_j(s)$ represent the expected value of the log of the contractual wage rate of the j^{th} group of workers at the start of indexation, and let $E \log p(s)$ represent the log of the expected aggregate price level at the start of indexation.³¹ For the j^{th} group of workers, the appropriate derivative of wages with respect to prices at the start of indexation is α_j , where

$$\log \alpha_j = \log e^*_{j} + E \log w_j(s) - E \log p(s) \quad ,$$

and e^*_{j} is the elasticity of wages with respect to prices predicted for the j^{th} group by the model. Let $\{\sigma_j\}$ represent a set of weights based on relative employment shares, and assume that α is chosen according to:

$$\log \alpha = \sum_j \sigma_j \log \alpha_j.$$

Substituting for the definition of $\log \alpha_j$ and making use of the approximation $\log e^*_{j} = e^*_{j} - 1$, this assumption implies

$$(12) \quad \log \alpha = -E \log p(s) + \sum_j \sigma_j E \log w_j(s) + \sum_j \sigma_j (e^*_{j}-1).$$

Equation (12) relates the observed contractual parameter α to the wage rates and composition of the labor-force in the contract, and to the

³¹When indexation is delayed, one or other of the price level or the wage rate at the start of indexation is uncertain at the signing date.

theoretically predicted elasticities e^*_j . Define the average contractual wage rate by $w = \exp\{\sum_j \sigma_j \log w_j\}$, and define the observed elasticity of indexation of average wages t periods after the start of the contract by $e(t) = a p(t)/w(t)$. Using this notation, equation (12) can be written as:

$$(13) \quad \log e(s) = \sum_j \sigma_j (e^*_j - 1) \quad .$$

The left hand side variable represents a useful characterization of wage escalation provisions in contracts with non-proportional indexation, non-contingent deferred wage increases, and multiple wage rates. Since the parameters r_1 , r_2 and r_3 are in principle measurable, and the coefficients c_1 , c_2 and c_3 are known functions of observable variables and taste parameters, equation (13) gives a concise summary of the empirical content of the theoretical model of indexation. Furthermore, the extent to which (13) provides an adequate description of the inter-contract dispersion in $e(s)$ gives a strong test of the theory. The balance of this section makes use of equation (13) to test the theoretical model of indexation against individual contract data.

b) Preliminary Data Analysis

The contract sample consists of 189 indexed contracts written between 1968 and 1975 covering 500 workers or more in the Canadian manufacturing sector. Contracts from non-manufacturing industries and contracts with proportional escalation clauses are excluded from the sample.³² In addition, a small number of contracts from the "miscellaneous manufacturing" industry classification are excluded for lack of meaningful industry data.

Information at the contract level includes the wage rate at each date over the life of the contract. Both the base wage rate and the highest wage rate covered by the contract are available.³³ Denote the base wage rate t periods after the start of the contract by $w_1(t)$, and denote the highest or "skilled" wage rate by $w_2(t)$. Contract-specific employment data are unavailable and skill composition in a particular contract must be inferred from the corresponding industry-level data. Let σ denote the proportion of skilled workers in the contract (according to the industry-level data) and define the average contractual wage rate as

$$w(t) = \exp[(1-\sigma)\log w_1(t) + \sigma \log w_2(t)] .$$

³²Detailed industry information is largely unavailable for non-manufacturing industries.

³³In most manufacturing contracts, the highest wage rate is earned by skilled maintenance tradesmen, while the base wage rate is earned by janitors or sweepers.

The marginal elasticity of average contract wages is defined accordingly. For comparative purposes, the data analysis is presented both in terms of $e(s)$, the elasticity at the start of indexation in the contract, and $e(0)$, the elasticity relative to wage rates and prices at the signing date of the contract.

For lack of firm-specific information on the variables identified by the theoretical model, any test of the model must rely on the maintained hypothesis that contracts within each 3 digit industry are essentially identical. On the other hand, a more natural assumption is that the contracts signed by each bargaining pair are identical, at least in terms of their desired elasticities of indexation. The hypothesis that contracts in each industry have the same elasticity of indexation, (apart from an error term) can be tested by comparing the residual variance from a regression of measured indexation elasticities on industry fixed effects with the residual variance from a regression on bargaining pair fixed effects.³⁴

Results for this procedure are reported in Table 1. Lines (5) and (6) of the Table give the F-ratios for the comparison of bargaining-pair and 3 digit industry fixed effects. In the fifth row, the test statistics are

³⁴If y_{ijk} represents the log of the elasticity of indexation in the k^{th} contract signed by the j^{th} bargaining group in the i^{th} industry, then a natural scheme for y_{ijk} is

$$y_{ijk} = y_i + y_{ij} + \varepsilon_{ijk} ,$$
where y_i represents the industry mean elasticity, y_{ij} has the interpretation of a fixed effect for the j^{th} bargaining group in the i^{th} industry, and ε_{ijk} is an error term. The test is simply $y_{ij} = 0$.

Table 1

Analysis of Variance of Escalation Elasticities^a

	Elasticity Measure ^b					
	e(0)	e(s)	e(0)	e(s)	e(0)	e(s)
1. Additional Fixed Effects:	--	--	--	--	--	--
2. Bargaining Pair F.E.						
a) Residual S.E.	.122	.126	.116	.121	.124	.128
b) F-test for Ad'nl F.E.	--	--	1.95 (.079)	1.83 (.100)	0.41 (.746)	0.55 (.600)
3. Industry Excluding pairs with one contract						
a) Residual S.E.	.123	.127	.115	.120	.124	.128
b) F-test for Ad'nl F.E.	--	--	2.78 (.012)	2.61 (.018)	0.60 (.617)	0.68 (.567)
4. Industry Including pairs with one contract						
a) Residual S.E.	.140	.139	.129	.129	.141	.140
b) F-test for Ad'nl F.E.	--	--	4.20 (.000)	4.20 (.000)	0.12 (.948)	0.09 (.965)
5. F-test for Ind. F.E. vs. Bargaining Pair F.E. Excluding pairs w/ one contract	1.07 (.402)	1.04 (.434)	0.96 (.528)	0.93 (.564)	1.02 (.458)	1.00 (.481)
6. F-test for Ind. F.E. vs. Bargaining Pair F.E. Including pairs w/ one contract	1.54 (.048)	1.36 (.103)	1.43 (.179)	1.24 (.198)	1.51 (.048)	1.35 (.112)

^aMarginal significance levels in parenthesis.

^bDependent variable is log of indicated elasticity measure. The sample mean of log e(0) is -.22 with a standard deviation of 0.19. The sample mean of log e(s) is -.22, with a standard deviation of 0.19.

reported for the subset of contracts written by parties with two or more contracts in the sample.³⁵ Test statistics for the entire sample of contracts are reported in the sixth line of the Table. Among the reported statistics, only one is significant at the 5 percent level. Substituting $e(o)$ for $e(s)$, or adding additional fixed effects to the regression, tends to increase the probability of accepting the null hypothesis. Overall, there is very little evidence from the Table that the within-industry variation in escalation elasticities is due to systematic bargaining-pair effects.

On the other hand, there is a fairly strong tendency for elasticities of indexation to vary systematically by the signing data of the contract.³⁶ This fact is only slightly less evident in the regressions that control for bargaining pair fixed effects (line (2b) of Table 1). In contrast to these results, there is no evidence of significant contract length fixed effects in measured elasticities of indexation, or alternatively, of significant fixed effects associated with any of major unions in the Canadian

³⁵Of the 189 contracts, 108 contracts were written by bargaining pairs with two or more contracts in the sample. In analysing bargaining pair fixed effects for the entire sample, only these contracts contribute to the sum of squared residuals of the regression.

³⁶Relative to contracts signed in 1975, average escalation elasticities within each 3 digit industry were approximately 15 percent lower in contracts signed in 1968, 9 percent lower in contracts signed in 1969, 7 percent lower in contracts signed in 1972, and approximately the same in the other years. Essentially the same pattern of estimated signing date fixed effect emerges, controlling for bargaining pairs.

manufacturing sector.³⁷ Apart from an error term and signing date effects, contracts in each 3 digit manufacturing industry have essentially the same elasticities of indexation. In fact, by themselves, 3 digit industry fixed effects account for about 60 percent of the sample dispersion in escalation elasticities, while in combination with signing date fixed effects they account for roughly two-thirds of this dispersion.

Although far from conclusive, these facts suggest that a model of indexation based on systematic industry factors is potentially capable of explaining a substantial fraction of the inter-contract variation in measured escalation elasticities. How far the model of Section I. can succeed in this regard is pursued in the next sub-section.

c) Estimates of the Model

With the substitution of equations (7a) or (10) into (13), the log of the elasticity of indexation is expressed as a non-linear function of the labor demand parameters, β , η and μ , the signal extraction parameters r_1 , r_2 , and r_3 , the relative wage effect z , and the parameters of the marginal cost of employment function, ϕ and ε , in case of (7a), or the risk parameters d and γ in case of (10). For simplicity, the labor demand parameters are assumed to be the same for both skill groups. In addition,

³⁷Controlling for bargaining pairs, union fixed effects are redundant.

firms in each industry are treated as price-takers, with the implication that $\mu = \beta + \eta$. Assuming a Cobb-Douglas technology, β and η can be estimated directly from cross-sectional data on the shares of labor and materials costs in industry shipments.³⁸ Using this procedure, the contract weighted average estimate of β for 44 industries is 1.67, while the average estimate of η is 2.32.

For purposes of the model of wage indexation, the joint distribution of output and input prices, alternative real wages, and the consumer price index is summarized by the parameters r_1 , r_2 and r_3 , which represent the population regression coefficients of innovations in each of the first three series on innovations in the latter series. In lieu of observations on alternative wages, the coefficient associated with the alternative wage series is treated as a fixed parameter for each skill group. On the other hand, differences across industries in the nature and composition of outputs and inputs are a potentially important source of inter-industry differences in escalation provisions. Assume that $\log \theta_t$, $\log r_t$, and $\log p_t$ are generated by a vector autoregression of the form:

³⁸The share of labor in the value of shipments is $(\beta-1)/(\beta+\eta)$, while the share of materials is $\eta/(\beta+\eta)$.

$$(14a) \quad \log p_t = a_1 \log p_{t-1} + a_2 \log p_{t-2} + \dots + a_k \log p_{t-k} + v_t ,$$

$$(14b) \quad \log \theta_t = b_1 \log \theta_{t-1} + b_2 \log \theta_{t-2} + \dots + b_k \log \theta_{t-k} + \\ c_1 \log p_{t-1} + c_2 \log p_{t-2} + \dots + c_k \log p_{t-k} + u_{1t} ,$$

$$(14c) \quad \log r_t = e_1 \log r_{t-1} + e_2 \log r_{t-2} + \dots + e_k \log r_{t-k} + \\ f_1 \log p_{t-1} + f_2 \log p_{t-2} + \dots + f_k \log p_{t-k} + u_{2t} ,$$

where v_t , u_{1t} , and u_{2t} are serially uncorrelated and jointly normally distributed. In this framework, the coefficients r_1 and r_2 have the simple interpretation of elements of the covariance matrix of the innovations v_t , u_{1t} , u_{2t} , with $r_1 = \text{cov}(u_{1t}, v_t) / \text{var}(v_t)$, and $r_2 = \text{cov}(u_{2t}, v_t) / \text{var}(v_t)$.

A number of alternative strategies are available for the estimation of r_1 and r_2 . Perhaps the simplest procedure is to estimate equations (14) by least squares and then perform the auxiliary regressions

$$(15a) \quad \tilde{u}_{1t} = r_1 \tilde{v}_t + \xi_{1t} ,$$

and

$$(15b) \quad \tilde{u}_{2t} = r_2 \tilde{v}_t + \xi_{2t} ,$$

using the estimated residuals \tilde{v}_t , \tilde{u}_{1t} , and \tilde{u}_{2t} . Since the vector autoregression contains exclusion restrictions, however, least squares

estimation of equations (14) is not fully efficient. As an alternative to least squares, a systems estimation scheme (maximum likelihood or a two-step procedure) can be applied to equations (14), and r_1 and r_2 can be obtained directly from the estimated covariance matrix of the system.³⁹

Industry-specific estimates of r_1 and r_2 were obtained by applying (14) and (15) to annual observations on industry selling prices (deflated by the consumer price index) and implicit prices of intermediate inputs (similarly deflated) by industry for the period 1963-1979.⁴⁰ Across industries, the correlation of r_1 and r_2 is strongly positive.⁴¹ The (contract weighted) averages of r_1 and r_2 are .27 and .28 respectively. However, there is somewhat greater dispersion in r_2 than r_1 . Higher values of both coefficients are estimated for food processing, textiles, and pulp and paper industries, whereas lower values of both coefficients are estimated for machinery industries, aircraft and automobile parts and assembly industries and electrical products industries.

Industry-specific estimates of the union markup of unskilled wage rates (z_1) were obtained by comparing wage rates of non-production laborers in

³⁹In fact, comparisons of estimates of r_1 and r_2 from equations (15) and from a two-step procedure reveal only minor discrepancies.

⁴⁰The choice of annual observations corresponds naturally to a model of indexation in two year contracts. Strictly speaking, the model of Section I. is appropriate only in the second year of a three year contract. For simplicity, the distinction between contracts of different lengths will be ignored.

⁴¹The contract weighted correlation of r_1 and r_2 is 0.76.

each industry at a specific location with the average wage rate of non-union laborers at the same location.⁴² On the other hand, the union markup of skilled wage rates is assumed to be constant across industries and equal to 1.05. This approximation is roughly consistent with the smaller union differentials for skilled as opposed to unskilled workers observed in U.S. micro data,⁴³ and with estimates of the union differential across industries for selected skilled occupations in Canada. As expected, the estimated union differential for unskilled workers is larger in soft drink, brewery, tobacco, aircraft, and motor vehicle industries. On the other hand, the differential is smaller in textile and clothing industries and negative in the furniture industry. The estimated differentials are probably biased by the failure to control for labor force quality by industry. Presumably, however, the general pattern of the differentials is correct.

For the model of indexation with risk neutrality, the remaining parameters of the escalation function are the elasticities of the opportunity cost of labor functions of unskilled and skilled workers -- ϵ_1 ,

⁴²Micro-data studies of union relative wage effects are unavailable for Canada. If the logarithm of the average wage rate in an industry is a weighted average of the non-union and union wage rates, and if the non-union wage rate is constant across industries, then the industry-specific union markup is the difference between the log of the industry average wage and the log of the non-union wage across all industries, divided by the percentage of unionized workers in the industry.

⁴³See for example Ashenfelter[2].

and ε_2 respectively.⁴⁴ These two, together with skill group specific estimates of the correlation of alternative real wages with the consumer price index (r_{31} and r_{32}) are taken as fixed parameters in the regression equation composed of (13) and (7a). For the model of indexation with risk aversion, the parameters of the regression include γ , the index of owners' risk aversion, d_1 and d_2 , the indexes of risk aversion of unskilled and skilled workers respectively, and the correlations r_{31} and r_{32} . Estimation results for the two alternative versions of the model are summarized in Tables 2a and 2b. For each version, estimates are reported both including and excluding contract signing date fixed effects in the regression, and for two measures of the elasticity of escalation of average contractual wage rates.

Turning first to the results in Table 2a for the model described by equations (10) and (13), the similarities of the parameter estimates across all four columns of the Table are immediately apparent. The estimates of the index of relative risk aversion of owners are small and positive, but statistically different from zero in every case. The estimates of d_1 are an order of magnitude larger and also statistically different from zero, while the estimates of d_2 are negative but not statistically different from zero in any of the regressions. The estimated correlations of short term movements in the alternative real wage rates of skilled and unskilled

⁴⁴The parameter ϕ is not identifiable if r_3 is unknown, and $\phi_1 = \phi_2 = 1$ is assumed.

Table 2a

Estimates of Equation (13):

Risk Averse Case

	Dependent Variable:			
	e(0)		e(s)	
Signing Date Fixed Effects Included:	NO	YES	NO	YES
Parameter: ¹				
γ	.097 (.032)	.066 (.032)	.084 (.035)	.071 (.036)
d_1	4.71 (1.72)	4.99 (1.51)	5.08 (1.92)	5.37 (1.75)
d_2	-.47 (.43)	-.40 (.60)	-.43 (.47)	-.34 (.65)
r_{31}	-.22 (.10)	-.23 (.10)	-.20 (.10)	-.20 (.10)
r_{32}	-.34 (.15)	-.29 (.16)	-.37 (.17)	-.33 (.18)
Residual S.E.	.171	.160	.170	.158
R^2	.19	.31	.20	.32
Likelihood Ratio vs. Ind. Fixed Effects (sig. level) ²	121.7 (.001)	127.3 (.001)	123.1 (.001)	128.6 (.001)

¹Standard Errors in parenthesis.

²The reported significance level is an upper bound on the actual significance level of the test.

workers with movements in the consumer price index are both negative. The correlation of skilled wage rates with the C.P.I. is somewhat more negative, indicating less nominal wage flexibility in the market for skilled labor.

The signs and magnitudes of these parameter estimates imply that the elasticity of the wage indexation schedule is determined by considerations of risk sharing between owners, unskilled workers, and skilled workers. According to the estimates, the desired wage escalator transfers some of the risk of input and output market price shocks from the owners of the firm to the skilled workers in the contract. Indexation provisions in contracts dominated by unskilled workers are largely unaffected by the characteristics of the input and output price shocks faced by the firm. For representative values of the industry-specific parameters, the coefficient of r_1 in the equation for the elasticity of indexation of unskilled workers is in the range of .01 - .03, while the coefficient of r_2 is in the range of (-.005 - -.015). In the corresponding equation for the elasticity of indexation of skilled workers, however, the coefficient of r_1 is much larger (between .15 and .20), and the coefficient of r_2 is correspondingly larger in absolute value (between -.08 and -.15).⁴⁵ Under the interpretation of the model, the cross-industry correlation of the elasticity of indexation of average wage rates with r_1 and r_2 is attributed

⁴⁵For example, with $\beta = 1.58$, $\eta = 2.13$, $z_1 = 1.15$, and with the parameter estimates in the fourth column of Table 2a, $c_1 = .03$, $c_2 = -.02$, $c_3 = .42$ for unskilled workers, while $c_1 = .20$, $c_2 = -.11$, and $c_3 = 1.30$ for skilled workers.

to the desired responsiveness of skilled wage rates to conditional inferences of output prices and non-labor input prices, given the aggregate price index.

While the parameter estimates of the model composed of (10) and (13) are relatively precise, the explanatory power of the resulting regression is not overwhelming. Excluding contract signing date fixed effects, the regression explains about 20 percent of the dispersion in the log of the elasticity of indexation of average wages. In combination with signing date fixed effects, the proportion of explained variance is approximately 30 percent. The last line of Table 2a reports likelihood ratio statistics for the model relative to an industry fixed effects regression. Assuming that the model is correct, these test statistics are asymptotically distributed as χ^2 variates with degrees of freedom not greater than the number of estimable industry fixed effects.⁴⁶ Even with this upper bound

⁴⁶The relation between the fixed effects regression and the non-linear model can be described as follows. The fixed effects regression (FE) gives:

$$y_{ij} = \delta_i + \varepsilon_{ij},$$

where y_{ij} is the elasticity of indexation of the j^{th} contract in the i^{th} industry, δ_i is the industry fixed effect, and ε_{ij} is an error term. The non-linear model (NL) gives:

$$y_{ij} = f(x_i, \beta) + \varepsilon_{ij},$$

where x_i is a vector of exogenous variables specific to the i^{th} industry, and β is a vector of parameters of the non-linear model. The restrictions imposed by NL on FE are

$$\begin{aligned} \delta_1 &= f(x_1, \beta), \\ \delta_2 &= f(x_2, \beta), \dots \end{aligned}$$

If NL is correct, δ lies in the range of a mapping from R^p to R^N , where p is the dimension of β , and N is the number of industries. Elements of the range of this mapping satisfy a set of at most N independent restrictions.

on the degrees of freedom, however, the marginal significance levels of the test statistics are less than one percent in all four columns of the Table. These findings provide strong evidence that the industry pattern of the variables β , η , z_1 , σ , r_1 and r_2 cannot fully account for the inter-industry distribution of elasticities of indexation.

A summary of results for the risk neutral version of the model is presented in Table 2b. Again the parameter estimates show remarkable similarity across the columns of the Table. As in Table 2a, the estimates of r_{31} and r_{32} are negative and statistically different from zero, with r_{31} ranging from $-.11$ to $-.14$ and r_{32} ranging from $-.41$ to $-.55$. The estimates of ϵ_1 imply that the opportunity wage schedule for unskilled workers is almost perfectly elastic. According to these estimates, the desired elasticity of indexation of unskilled wage rates is independent of the nature of the shocks to input and output prices faced by the firm, and approximately equal to the coefficient r_{31} .⁴⁷ On the other hand, the opportunity cost schedule for skilled workers is upward sloping. As in the case of the risk-averse version of the model, the cross-sectional relationship between measured elasticities of indexation and the signal extraction coefficients r_1 and r_2 is attributed to the influence of skilled workers in the contract.

⁴⁷The coefficient of r_1 for unskilled workers is $c_1 = (\beta + \eta) / (\beta + \epsilon_1)$. For ϵ_1 of order 10^6 , this coefficient is approximately 0. A similar result obtains for the coefficient of r_2 . On the other hand, the coefficient of r_3 is $\epsilon_1 / (\beta + \epsilon_1) \cong 1$.

Table 2b

Estimates of Equation (13):

Risk Neutral Case

	Dependent Variable:			
	e(0)		e(s)	
Signing Date Fixed Effects Included:	NO	YES	NO	YES
Parameter ¹				
$\epsilon_1 \times 10^6$	9.69 (1720)	4.63 (927)	10.3 (2070)	2.58 (403)
ϵ_2	9.55 (2.55)	12.44 (3.92)	9.50 (2.46)	12.41 (3.88)
r_{31}	-.14 (.04)	-.13 (.05)	-.12 (.05)	-.11 (.05)
r_{32}	-.52 (.11)	-.41 (.11)	-.55 (.11)	-.44 (.11)
Residual S.E.	.175	.167	.175	.166
R^2	.14	.25	.15	.26
Likelihood Ratio Linear term in z_1 (sig. level) ¹	8.26 (.001)	12.02 (.001)	8.02 (.001)	11.76 (.001)
Likelihood Ratio vs. Ind. Fixed Eff. (sig. level) ²	134.8 (.001)	143.1 (.001)	134.4 (.001)	144.2 (.001)

¹Standard Errors in parenthesis.

²The reported significance level is an upper bound on the actual significance level of the test.

A comparison of the estimated standard errors in the fifth line of Tables 2a and 2b reveals that the risk averse version of the model fits somewhat better than the alternative. The difference is due in part to the impact of measured union wage differentials on the elasticity of indexation. According to the risk-neutral version of the theory, the elasticity of indexation is independent of z_1 . However, as the test statistics in the second last row of Table 2b indicate, the addition of a linear term in z_1 to the regression composed of (7a) and (13) leads to a significant improvement in the likelihood of the sample. Contrary to the model, the residual from the risk-neutral specification is significantly correlated with the index of union wage differentials.

As with the risk averse version of the model, a comparison of the fit of the regression composed of equations (7a) and (13) with an industry fixed effects regression indicates that the former cannot account fully for the industry pattern of escalation elasticities. Given the results in Table 2a, this is a predictable short-coming of the model.

A more direct comparison of the two versions of the theoretical model is provided by a non-nested hypothesis testing procedure. Following Davidson and Mackinnon[3], suppose that the risk-averse variant of the model is represented by:

$$(16a) \quad \log e(s) = f(x_a, \delta_a) + u_a \quad ,$$

while the risk neutral variant is given by:

$$(16b) \quad \log e(s) = g(x_n, \delta_n) + u_n \quad .$$

Here, x_a and x_n are model-specific vectors of industry variables, γ_a and γ_n are parameter vectors, and u_a and u_n are normally distributed errors. The functional forms implied by (10) and (13) are subsumed in $f(x_a, \gamma_a)$, while those implied by (7a) and (13) are represented by $g(x_n, \gamma_n)$.⁴⁸

Davidson and Mackinnon propose the following test of (16a) against (16b). First, estimate (16b) by least squares, and form the predictions $g(x_n, \delta_n)$. Then, estimate the artificially nested model

$$(17) \quad \log e(s) = (1-\alpha) f(x_a, \delta_a) + \alpha g(x_n, \delta_n) + u'_a \quad .$$

They show that under the null hypothesis that (16a) is the true model generating the data, the least squares estimate of α in equation (17) has an asymptotic normal distribution with mean zero. Furthermore, the standard error of this distribution is the probability limit of the estimated standard error from the regression. These facts permit a remarkably simple test of (16a) against the specific alternative of (16b), based on the t-statistic for α from the artificially nested regression.

⁴⁸In fact, x_a differs from x_n by the addition of the variable z_1 . The elements of δ_a are $\gamma, d_1, d_2, r_{31}, r_{32}$. The elements of δ_n are $\epsilon_1, \epsilon_2, r_{31}, r_{32}$.

Table 3

Non-Nested Tests of Risk Averse and
Risk Neutral Versions of the Model¹

	Maintained Assumption:			
	Risk Averse		Risk Neutral	
	(1)	(2)	(3)	(4)
Signing Date Fixed Effects Included:	NO	YES	NO	YES
Estimate of α	-.57	-1.16	1.01	1.01
(asymptotic t- ratio)	.87	1.51	5.87	5.66

¹The dependent variable in each case is $\log e(s)$.

Results for this testing procedure are presented in Table 3.⁴⁹ For simplicity, only the estimates of α and their asymptotic t-ratios are reported. The first two columns give the test results with the risk-averse model as the maintained hypothesis and the risk-neutral model as the alternative. In the last two columns, these roles are reversed. The tests are performed both excluding and including contract signing date fixed effects from the maintained and alternative models.

The estimated coefficients and t-ratios suggest that while the risk averse model cannot be rejected against the risk neutral version, the opposite conclusion is not true. In fact, the estimate of α is almost exactly 1.0 in the last two columns of the Table. Although the distribution of α is unknown when the maintained hypothesis is incorrect and the alternative is true, it is clear that estimates of α around 1.0 might be expected under such circumstances. Thus the non-nested hypothesis testing procedures provide further against the risk neutral specification of the model, and in favor of the risk averse version with a constant opportunity cost of employment.⁵⁰

⁴⁹The dependent variable in each case is $\log e(s)$. Results for $\log e(o)$, the log of the elasticity of indexation relative to real wage rates at the start of the contract, are essentially identical to those reported in Table 3.

⁵⁰While these two models are not nested, the risk neutral version with ε_1 and ε_2 large is equivalent in the limit to a risk averse version in which $\gamma = d_1 = d_2 = 0$. Since the estimate of ε_1 is essentially infinite, and the estimate of ε_2 is large, the estimated risk neutral version is fairly close to a restricted version of the alternative.

Conclusions

This paper has presented a theoretical model of wage indexation and described the results of fitting the model to a sample of escalated contracts from the Canadian manufacturing sector. These results are moderately supportive of the theory, although they indicate some major difficulties with the model itself and with its empirical implementation. Specifically, while the contract data are not grossly inconsistent with the implications of the theory, it is clear that the theory does not by itself provide a complete description of the data.

In the first instance, observed indexation provisions are considerably more complex than allowed by the the theoretical set-up. Any empirical model of automatic wage adjustment in long term labor contracts must ultimately come to grips with non-proportional indexation, contingent and non-contingent wage increases, and shared indexation provisions. For the most part, these aspects of observed escalation clauses have been ignored, and cost of living allowance clauses have been summarized in terms of a single number, based on the marginal elasticity of average contractual wage rates at the start of indexation. While the elasticity of indexation is of considerable practical interest, especially in a macroeconomic context, a complete model of wage escalation should provide insights into the other structural aspects of indexed contracts.

Within the narrow domain of modelling the contract distribution of

elasticities of indexation, the theoretical framework is still not rich enough to fully describe the data. For example, the signing date of the contract is a significant determinant of the elasticity of indexation, even controlling for the effects of specific bargaining pairs. While in principle this fact could be explained by temporal instability in the parameters of the theoretical model, a test of this hypothesis will require considerable further research. At the same time, the variables identified by the model as determinants of the elasticity of indexation do not completely account for the dispersion in industry-average elasticities of indexation. However, whether this is a result of measurement error in these variables or an inadequacy of the model is not determinable. More sophisticated measures of the variables in the model would undoubtedly increase the level of confidence in the empirical findings.

In spite of these shortcomings, the model does provide several insights into the determinants of the elasticity of indexation in observed labor contracts. Each of the variables in the model, including the elasticities of labour demand with respect to real wages and non-labor input prices, the proportion of skilled workers in the production labor force of the firm, the relative union wage gap for unskilled workers, and the correlations of short term movements in output and input prices with the C.P.I., have a significant influence on the elasticity of the contractual wage escalator. Furthermore, the directions and magnitudes of their influences are consistent with the implications of the theoretical model. In particular,

they are consistent with a model of risk sharing between owners and workers in which owners receive partial insurance from the shocks to input and output prices, at the same time as workers receive partial insurance from shocks to alternative real wage rates.

While it would be premature to claim that such a model forms the basis for the contractual arrangements observed in a large sector of the North American labor market, the findings in the paper suggest that there may be some empirical support for this contention. In view of importance of long term contracts for the performance of the economy and economists' policy prescriptions, the goal of obtaining an empirically acceptable theoretical interpretation of labor contracts remains an important one.

References

- [1] Altonji, J. and O. Ashenfelter. "Wage Movements and the Labor Market Equilibrium Hypothesis". Economica, vol. 47, no. 187 (August 1980), pp. 217-245.
- [2] Ashenfelter, O. "Union Relative Wage Effects: New Evidence and a Survey of Their Implications for Wage Inflation". In R. Stone and N. Peterson (eds.) Econometric Contributions to Public Policy. London: MacMillan, 1978.
- [3] Azariadis, Costas. "Implicit Contracts and Underemployment Equilibria". Journal of Political Economy, vol. 83, no. 6 (December 1975), pp. 1183-1202.
- [4] Blanchard, O.-J. "Wage Indexation Rules and the Behaviour of the Economy". Journal of Political Economy, vol. 87, no. 4, (August 1979), pp. 798-815.
- [5] Blinder, A.S. "Indexing the Economy Through Financial Intermediation". In K. Brunner and A.H. Meltzer (eds.) Stabilization of the Domestic and International Economy. Carnegie-Rochester Conference Series on Public Policy No. 5. Amsterdam: North Holland, 1977.
- [6] Breusch, T.S. and A.R. Pagan. "A Simple Test for Heteroscedasticity and Random Coefficient Variation". Econometrica, vol. 47, no. 5 (September 1979), pp. 1287-1294.
- [7] Calvo, G.E and E.S. Phelps. "Employment Contingent Wage Contracts". In K. Brunner and A.H. Meltzer (eds.) Stabilization of the Domestic and International Economy. Carnegie-Rochester Conference Series on Public Policy No. 5. Amsterdam: North Holland, 1977.
- [8] Card, D. "Cost of Living Escalators in Major Union Contracts". Industrial and Labor Relations Review, forthcoming, 1982.
- [9] Cukierman, A. "The Effects of Wage Indexation on Macroeconomic Fluctuations: A Generalization". Journal of Monetary Economics, vol. 6, no. 2 (April 1980), pp. 147-170.
- [10] Davidson, R. and J. Mackinnon. "Several Tests for Model Specification in the Presence of Alternative Hypotheses". Econometrica, vol. 49, no. 3 (May 1981), pp. 781-793.

- [11] Dunlop, J.T. Wage Determination Under Trade Unions. New York: MacMillan, 1944.
- [12] Fischer, S. "Wage Indexation and Macroeconomic Stability". In K. Brunner and A.H. Meltzer (eds.) Stabilization of the Domestic and International Economy. Carnegie-Rochester Conference Series on Public Policy No. 5. Amsterdam: North Holland, 1977.
- [13] Garbarino, J. Wage Policy and Long Term Contracts. Washington: The Brookings Institution, 1962.
- [14] Gray, JoAnna. "Wage Indexation: A Macroeconomic Approach". Journal of Monetary Economics, vol. 2, no. 2 (April 1976), pp. 221-236.
- [15] Gray, JoAnna. "On Indexation and Contract Length". Journal of Political Economy, vol. 86, no. 1 (February 1978), pp. 1-18.
- [16] Grossman, S.J. and O. Hart. "Implicit Contracts, Moral Hazard and Unemployment". American Economic Review, vol. 71, no. 2 (May 1981), pp. 301-307.
- [17] Hall, R.E. and E. Lazear. "The Excess Sensitivity of Layoffs and Quits to Demand". National Bureau of Economic Research Document No. 114, February 1982.
- [18] Hall, R.E. and D. Lilien. "Efficient Wage Bargains Under Uncertain Supply and Demand". American Economic Review, vol. 69, no. 4, pp. 868-879.
- [19] LeRoy, D.R. "Scheduled Wage Increases and Cost of Living Provisions in 1982". Monthly Labor Review, vol. 105, no. 1 (January 1982), pp. 16-20.
- [20] Stigler, G.J. and J. Kindahl. The Behaviour of Industrial Prices. New York: National Bureau of Economic Research, 1970.
- [21] Taft, P. "Collective Bargaining Before the New Deal". In H. A. Millis (ed.) How Collective Bargaining Works. New York: The Twentieth Century Fund, 1945.
- [22] Taylor, John B. "Union Wage Settlements During a Disinflation". Princeton University Unpublished Manuscript, February 1982.

Appendix A

This Appendix presents a general derivation of the elasticity of indexation when both owners and workers are risk neutral. Suppose that the preferences of a typical worker are represented by:

$$A.1 \quad 1/L^0 \left(\frac{w}{p} L - c(L,a) \right),$$

where $c(L,a)$ has the interpretation of the cost of employing L workers when the alternative real wage is a . If $c(L,a) = \frac{\epsilon}{\epsilon-1} a^\phi L^{\epsilon-1/\epsilon}$, then the marginal cost of employing one more unit of labor in the contract is $a^\phi L^{1/\epsilon}$. Suppose that the profit function of the firm is given by

$$A.2 \quad \Pi(\theta, r, w, p) = \frac{1}{\beta-1} \theta^\mu (w/p)^{1-\beta} r^{-\eta}.$$

Differentiating with respect to the real wage rate, the employment demand schedule is:

$$L = \theta^\mu (w/p)^{-\beta} r^{-\eta}.$$

With some simplifications, the first order condition for $w(p)$ is given by:

$$A.3 \quad \left(\frac{w}{p} \right)^{(\epsilon+\beta)/\epsilon} = \left(\frac{\beta}{\beta+\eta L^{\phi-1}} \right) E(a^\phi | p) E(\theta^{\mu/\epsilon} | p) E(r^{-\eta/\epsilon} | p).$$

Taking the limit as $\epsilon \rightarrow \infty$, with $\phi = 1$, this expression reduces to equation (6) of the text. Taking logarithms of A.3 and rearranging slightly, we obtain:

$$A.4 \quad \log\left(\frac{w}{p}\right) = \left(\frac{\epsilon}{\epsilon+\beta}\right) \log\left(\frac{\beta}{\beta+\eta L^{\phi-1}}\right) + \log E(a^{\phi\epsilon/(\epsilon+\beta)} | p) \\ + \log E(\theta^{\mu/(\epsilon+\beta)} | p) + \log E(r^{-\eta/(\epsilon+\beta)} | p),$$

where use is made of the fact that

$$\{E(x)\}^\delta = E(x^\delta)$$

if x is lognormally distributed. Finally, using the fact that

$$\frac{\delta \log E(y^\delta | x)}{\delta \log x} = \delta \frac{\delta E(\log y | \log x)}{\delta \log x} ,$$

if x and y have a joint lognormal distribution, and the expressions for the conditional expectations presented in Appendix B, the elasticity of indexation is:

$$\frac{d \log(w/p)}{d \log p} = \frac{\mu}{\varepsilon + \beta} r_1 - \frac{\eta}{\varepsilon + \beta} r_2 + \frac{\phi \varepsilon}{\varepsilon + \beta} r_3 .$$

Appendix B

This Appendix presents a derivation of equation (9) in the text. The first order condition (4) requires evaluation of $E(\delta U/\delta w | p)$ and $E(\delta v(\Pi)/\delta w | p)$. Substituting from the definitions:

$$U = L/L^0 \{ u(w/p) - u(a) \} + u(a) ,$$

$$u(x) = \frac{1}{1-d} x^{1-d} ,$$

and

$$L = \theta^\mu (w/p)^{-\beta} r^{-\eta} ,$$

the first of these expressions can be written as:

$$\begin{aligned} \text{B.1} \quad E(\delta U/\delta w | p) &= \frac{1}{pL^0} \left\{ \frac{1-\beta-d}{1-d} \right\} (w/p)^{-\beta-d} E(r^{-\eta} \theta^\mu | p) \\ &+ \frac{1}{pL^0} \left\{ \frac{\beta}{1-d} \right\} (w/p)^{-\beta-1} E(r^{-\eta} \theta^\mu a^{1-d} | p) . \end{aligned}$$

Using the definitions

$$\Pi(\theta, r, w, p) = \frac{1}{\beta-1} \theta^\mu (w/p)^{1-\beta} r^{-\eta} ,$$

and

$$v(\Pi) = \frac{1}{1-\gamma} \Pi^{1-\gamma} ,$$

the second of these is given by:

$$\text{B.2} \quad E(\delta v(\Pi)/\delta w | p) = -\frac{1}{p} \left\{ \frac{1}{1-\beta} \right\}^{-\gamma} (w/p)^{-\beta-\gamma(1-\beta)} E(\theta^{\mu(1-\gamma)} r^{\eta(\gamma-1)} | p) .$$

Substituting B.1 and B.2 into the first order condition gives:

$$\begin{aligned} \text{B.3} \quad &\frac{1-\beta-d}{1-d} (w/p)^{-\beta-d} E(r^{-\eta} \theta^\mu | p) + \frac{\beta}{1-d} (w/p)^{-\beta-1} E(r^{-\eta} \theta^\mu a^{1-d} | p) \\ &= hL^0 \left(\frac{1}{1-\beta} \right)^{-\gamma} \left(\frac{w}{p} \right)^{-\beta-\gamma(1-\beta)} E(\theta^{\mu(1-\gamma)} r^{\eta(\gamma-1)} | p) . \end{aligned}$$

If $\log x$ and $\log y$ have a joint normal distribution with mean vector 0, $\text{var}(\log x) = \sigma_x^2$, $\text{var}(\log y) = \sigma_y^2$, and $\text{cov}(\log x, \log y) = \rho \sigma_x \sigma_y$, then:

$$E(y^\delta | x) = \exp\left\{ \delta \rho \frac{\sigma_x}{\sigma_y} \log x + \frac{1}{2} \delta^2 \sigma_y^2 (1-\rho^2) \right\} .$$

Using this general expression and the notation in the text, we obtain:

$$\begin{aligned} \text{B.4} \quad E(\theta^\mu | p) &= \exp\left\{ \mu r_1 \log(p/\hat{p}) + \frac{1}{2} \mu^2 \sigma_1^2 (1-\rho_1^2) \right\} \\ &= \exp\{ m_1 + s_1 \} , \end{aligned}$$

$$\begin{aligned} E(r^{-\eta} | p) &= \exp\{-\eta r_2 \log(p/\hat{p}) + \frac{1}{2} \eta^2 \sigma_2^2 (1-\rho_2^2) \} \\ &= \exp\{ m_2 + s_2 \} , \end{aligned}$$

$$\begin{aligned} E(a^{1-d} | p) &= \exp\{(1-d)r_3 \log(p/\hat{p}) + \frac{1}{2}(1-d)^2 \sigma_3^2 (1-\rho_3^2)\} \\ &= \exp\{ m_3 + s_3 \} . \end{aligned}$$

Substituting these into B.3 and simplifying, we obtain:

$$\begin{aligned} \text{B.5} \quad & \left(\frac{1-\beta-d}{1-d}\right) \left(\frac{w}{p}\right)^{-d} \exp\{m_1+m_2+s_1+s_2\} + \frac{\beta}{1-d} \left(\frac{w}{p}\right)^{-1} \exp\{m_1+m_2+m_3+s_1+s_2+s_3\} \\ &= hL^0 \left(\frac{1}{1-\beta}\right) \left(\frac{w}{p}\right)^{-\gamma(1-\beta)} \exp\{(1-\gamma)(m_1+m_2)+(1-\gamma)^2(s_1+s_2)\} \end{aligned}$$

If $\gamma=0$ it is clear that the terms in m_1 , m_2 , s_1 , and s_2 cancel in equation B.5, and the wage function is independent of r_1 and r_2 . Isolating the term $hL^0 \left(\frac{1}{1-\beta}\right)^\gamma$ on the right hand side of B.5 yields:

$$\begin{aligned} \text{B.5a} \quad & \left(\frac{1-\beta-d}{1-d}\right) \left(\frac{w}{p}\right)^{\gamma(1-\beta)-d} \exp\{\gamma(m_1+m_2)+\gamma(2-\gamma)(s_1+s_2)\} \\ & + \frac{\beta}{1-d} \left(\frac{w}{p}\right)^{\gamma(1-\beta)-1} \exp\{\gamma(m_1+m_2)+m_3+\gamma(2-\gamma)(s_1+s_2)+s_3\} \\ & = hL^0 \left(\frac{1}{1-\beta}\right)^\gamma . \end{aligned}$$

Differentiating B.5a with respect to p gives:

$$\begin{aligned}
 \text{B.6} \quad \frac{1}{p} \left(\frac{dw}{dp} - \frac{w}{p} \right) \Delta &+ \frac{1}{p} \gamma (r_1 + r_2) \left(\frac{1-\beta-d}{1-d} \right) \left(\frac{w}{p} \right) \gamma^{(1-\beta)-d} \exp \{ \Psi \} \\
 &+ \frac{1}{p} (\gamma (r_1 + r_2) + r_3) \left(\frac{\beta}{1-d} \right) \left(\frac{w}{p} \right) \gamma^{(1-\beta)-d} \exp \{ \Psi + m_3 + s_3 \} \\
 &= 0 .
 \end{aligned}$$

where

$$\begin{aligned}
 \Delta &= \left(\frac{\gamma(1-\beta)-d}{(1-d)} \right) (1-\beta-d) \left(\frac{w}{p} \right) \gamma^{(1-d)-d-1} \exp \{ \Psi \} \\
 &+ \left(\frac{\gamma(1-\beta)-1}{1-d} \right) \beta \left(\frac{w}{p} \right) \gamma^{(1-\beta)-2} \exp \{ \Psi + m_3 + s_3 \} ,
 \end{aligned}$$

and

$$\Psi = \gamma(m_1 + m_2) + \gamma(2-\gamma)(s_1 + s_2) .$$

With some simplification, this expression yields equation (9) in the text.